Diff. Eqns $\qquad$ $D_2(H)$ 4^{th} Paper

LDECC

Q: Solve $(D^2+a^2)y = \sec ax$.

Soln

For CF, $D^2+a^2=0 \Rightarrow D = \pm a$

$\therefore$ CF $= c_1 \cos ax + c_2 \sin ax$.

For PI

$$PI = \frac{1}{D^2+a^2}\sec ax = \frac{1}{D^2-i^2a^2}\sec ax$$

$$\Rightarrow PI = \frac{1}{(D+ai)(D-ai)}\sec ax$$

$$= \frac{(D+ia)-(D-ia)}{2ai\,(D+ai)(D-ai)}\sec ax$$

$$\Rightarrow PI = \frac{1}{2ai}\left[\frac{1}{D-ai} - \frac{1}{D+ai}\right]\sec ax \quad \text{(1)}$$

$$\Rightarrow PI = \frac{1}{2ai}\frac{1}{(D-ai)}\sec ax - \frac{1}{2ai(D+ai)}\sec ax \quad (1)$$

Now, $\frac{1}{D-ai} \sec ax = \frac{1}{(D-ai)} \frac{1}{\cos ax}$

$$= \frac{1}{(D-ai)} \frac{e^{iax} \cdot e^{-iax}}{\cos ax}$$

$$= e^{iax} \int \frac{e^{-iax}}{\cos ax} dx = e^{iax} \int \frac{\cos ax - i \sin ax}{\cos ax} dx$$

$$= e^{iax} \int \left[1 - i \tan ax \right] dx$$

$$\Rightarrow \frac{1}{D-ai} \sec ax = e^{iax} \left(x + \frac{i}{a} \log \cos ax \right)$$

Similarly $\frac{1}{D+ai} \sec ax = e^{-iax} \left(x - \frac{i}{a} \log \cos ax \right)$

using the above two equs in eq. (1), we've

$$\Rightarrow PI = \frac{1}{2ai} e^{iax} \left(x + \frac{i}{a} \log \cos ax \right) - \frac{1}{2ai} e^{-iax} \left(x - \frac{i}{a} \log \cos ax \right)$$

$$\Rightarrow PI = \frac{x}{2ai} \left(e^{iax} - e^{-iax} \right) + \frac{1}{2a^2} \log \cos ax \left(e^{iax} + e^{-iax} \right)$$

$$= \frac{x}{2ai} \cdot 2i \sin ax + \frac{1}{2a^2} \log \cos ax \cdot 2 \cos ax$$

$$\Rightarrow PI = \frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax \log \cos ax$$

Hence, the complete soln is $y = CF + PI$.